Exam 01003 ReF24

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål
Hvert rigtigt svar giver 1 point
Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question
Every correct answer corresponds to 1 point
Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

A set consists of the following two vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \ , \ \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

Which of the below vectors must be added to the set such that it spans all of $\mathbb{R}^3$?

Vælg en svarmulighed

- ○ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- ○ $\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$
- ○ $\begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix}$
- ○ $\begin{bmatrix} 0 \\ -4 \\ 8 \end{bmatrix}$
- ○ There is no vector among the answer options that fulfills the requirement!

We are given the matrices $\mathbf{A}, \mathbf{C} \in \mathbb{R}^{2\times 2}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} \text{ and } \mathbf{C} = \begin{bmatrix} 4 & -4 \\ -4 & 6 \end{bmatrix}.$$

Which of the below $\mathbf{B}$ matrices is a solution to the matrix equation $\mathbf{A} \cdot \mathbf{B} = \mathbf{C}$?

Vælg en svarmulighed

- $\mathbf{B} = \begin{bmatrix} 0 & 2 \\ 2 & -3 \end{bmatrix}$
- $\mathbf{B} = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$
- $\mathbf{B} = \begin{bmatrix} 0 & -2 \\ -2 & 3 \end{bmatrix}$
- $\mathbf{B} = \begin{bmatrix} 1 & -2 \\ -2 & -3 \end{bmatrix}$
- No such matrix exists among the answer options!

We are given the matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ k & 0 & 3 \\ -2 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times3}.$$

For which value of $k \in \mathbb{R}$ is the determinant of the matrix equal to zero ($\det(\mathbf{A}) = 0$)?

Vælg en svarmulighed

- $k = -\frac{2}{3}$
- $k = -\frac{3}{2}$
- The wanted value does not exist among the answer options
- $k = \frac{3}{2}$
- $k = \frac{2}{3}$

A second-degree polynomial with real coefficients is given:

$$p(z) = az^2 - bz + c \quad , \quad z \in \mathbb{C}.$$

We are informed that the polynomial has the root $z_0 = 1 - 2i$. Furthermore, we are informed that $p(0) = 10$.

Which of the below answer options states the polynomial p ?

Vælg en svarmulighed

- $p(z) = 2z^2 - 4z + 10$
- $p(z) = z^2 - 2z + 5$
- $p(z) = -2z^2 + 4z - 10$
- $p(z) = -z^2 + 4z + 5$
- None of the answer options state a polynomial that fulfills the requirements!

Let $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 5 & 0 & -2 \end{bmatrix}$ be a matrix and let $\mathbf{b} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$ be a column vector.

Which of the below column vectors states the result of the matrix product $\mathbf{A} \cdot \mathbf{b}$?

Vælg en svarmulighed

- $\mathbf{x} = \begin{bmatrix} 6 \\ -16 \end{bmatrix}$
- $\mathbf{x} = \begin{bmatrix} 6 \\ 16 \end{bmatrix}$
- $\mathbf{x} = \begin{bmatrix} -14 \\ 1 \\ 3 \end{bmatrix}$
- $\mathbf{x} = \begin{bmatrix} -14 \\ -1 \\ 3 \end{bmatrix}$
- The result of the matrix product does not exist among the answer options!

A real system of differential equations is given by

$$\begin{bmatrix} f_1'(t) \\ f_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, \text{ where } \mathbf{A} \in \mathbb{R}^{2\times 2}.$$

We are informed that the general solution to the system is given by:

$$\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} e^{-2t} + k_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t},\ \ t, k_1, k_2 \in \mathbb{R}\,.$$

Which of the below options states an eigenvalue with associated eigenvector of the matrix $\mathbf{A}$?

Vælg en svarmulighed

- $\lambda = -2$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$
- $\lambda = -2$ and $\mathbf{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$
- $\lambda = -1$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- None of the answer options states an eigenvalue with associated eigenvector with the wanted properties!
- $\lambda = -1$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ be a matrix given by:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 1 \\ 0 & 8 & 4 \end{bmatrix}.$$

Let $L_{\mathbf{A}} : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map given by: $L_{\mathbf{A}}(\mathbf{v}) = \mathbf{A}\mathbf{v}$.

Which of the below options states the kernel of the map $L_{\mathbf{A}}$?

Vælg en svarmulighed

- None of the above answer options state the kernel of $L_{\mathbf{A}}$.
- $\ker(L_{\mathbf{A}}) = \text{span}(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix})$
- $\ker(L_{\mathbf{A}}) = \text{span}(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix})$
- $\ker(L_{\mathbf{A}}) = \text{span}(\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix})$
- $\ker(L_{\mathbf{A}}) = \text{span}(\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix})$