

Homework set 1

- a) Determine whether the following two logical propositions are logically equivalent:

$$(P \Leftrightarrow Q) \wedge P \quad \text{and} \quad P \Rightarrow Q.$$

- b) A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by the expression $f(x) = x^2 - 3x + 2 - 3 \cdot |x - 1|$.

1. Determine whether the function is injective.
2. Determine the image set of the function.

- c) We are given the following two subsets of the complex numbers:

$$A = \{z \in \mathbb{C} \mid |z - i| = 2\} \quad \text{and} \quad B = \{z \in \mathbb{C} \mid \operatorname{Re}(z) = 1\}.$$

1. Draw the sets A and B in the complex plane.
2. Determine $A \cap B$.

- d) Show that the complex number $(1 - i)^{80}$ is a real number.

- e) As usual the principal value of the argument of a complex number z is denoted by $\operatorname{Arg}(z)$. Determine whether the following propositions are true:

1. $\operatorname{Arg}(z) = \pi \Rightarrow z \in \mathbb{R}_{>0}$.
2. $z \in \mathbb{R} \Rightarrow \operatorname{Arg}(z) = 0$.

The homework must be uploaded to the **DTU Learn** module for the course via "Assignments".
Deadline is **Sunday 27 September at 23:55**.