Technical University of Denmark

Multiple-choice test exam, November 2023

Course name: Mathematics 1a (Polytechnical foundation) Course no. 01001 and 01003
Exam duration: 2 hours

Aid: All by DTU permitted aid.

"Weighting": All questions in this test exam are weighted equally. This part of the test exam constitutes 50% of the entire test exam.

Additional information: The questions are posed first in English, after that in Danish. All questions are multiple choice and no explanatory text or calculations will be taken into account. *Note: This test exam is a transcipt; the test exam will be digital and is to be answered on the DTU Digital Exam platform.*

Question 1

Given is the following system of linear equations over $\mathbb{R}$ in the three unknowns x_1, x_2, and x_3:

$$\begin{cases} x_2 + x_3 = 1 \\ x_1 - 2x_2 + x_3 = 4 \end{cases}.$$

Which of the below sets denotes the complete solution to the system?

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, t \in \mathbb{R}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

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Question 2

We are given:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Which of the following expressions is identical to the expression $(\mathbf{A} \cdot \mathbf{B})^T \cdot \mathbf{v}$?

$$\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 11 \\ 2 & 4 \\ -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

Question 3

We are given

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 9 \\ 1 & a & 1 \\ 0 & -1 & a \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

For which values of $a \in \mathbb{R}$ is matrix $\mathbf{A}$ invertible?

$a \in \mathbb{R} \setminus \{-2, 4\}$

$a \in \{-2, 4\}$

$a \in \mathbb{R}$

$a \in \mathbb{R} \setminus \{0\}$

$a \in \mathbb{R} \setminus \{0, 1\}$

Question 4

We are informed that $Q(z)$ is a polynomial of degree 2 with the roots 1 and 3. We are given

$$P(z) = Q(z) \cdot (z^3 - 5z^2 - z + 5),$$

and we are informed that $P(z)$ has the root $z = -1$. What are all roots and corresponding multiplicities of P?

$z_1 = -1$ with multiplicity 1, $z_2 = 3$ with multiplicity 1, $z_3 = 5$ with multiplicity 1, and $z_4 = 1$ with multiplicity 2.

$z_1 = -1$ with multiplicity 1, $z_2 = 1$ with multiplicity 1, and $z_3 = 3$ with multiplicity 1.

$z_1 = -1$ with multiplicity 2, $z_2 = 1$ with multiplicity 2, and $z_3 = 3$ with multiplicity 1.

$z_1 = -1$ with multiplicity 1, $z_2 = 3$ with multiplicity 1, $z_3 = 5$ with multiplicity 1, and $z_4 = 1$ with multiplicity 1.

$z_1 = 0$ with multiplicity 1, $z_2 = 1$ with multiplicity 1, $z_3 = 3$ with multiplicity 1, and $z_4 = 5$ with multiplicity 1.

Question 5

Let

$$\mathbf{B} = \begin{bmatrix} 4 & -1 & -1 \\ 6 & -3 & -1 \\ -6 & 5 & 3 \end{bmatrix} \in \mathbb{R}^{3\times3}.$$

Choose matrices Λ and $\mathbf{V}$ such that $\mathbf{V}^{-1}\mathbf{B}\mathbf{V} = \Lambda$.

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}, \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

No matrices **B** and Λ that fulfill the above relation exist.

Question 6

Let V be a subspace of $\mathbb{R}[Z]$, and let V be equipped with the ordered basis $\alpha = (1, Z, Z^2)$. We are given the following information about a linear map $f : V \to V$:

$$f(1) = 1 + Z,\ f(Z) = 1 - Z, \text{ and } f(Z^2) = Z + Z^2.$$

What are the eigenvalues of this map?

$1, \sqrt{2}, -\sqrt{2}$

$1, -1, 1$

$-1, -\sqrt{2}, \sqrt{2}$

$\sqrt{2}, -1, 1$

$-\sqrt{2}, 0, 1$

Question 7

A real system of differential equations is given by

$$\begin{bmatrix} f_1'(t) \\ f_2'(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}.$$

We are informed that $\begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$. What is the solution?

$f_1(t) = 3e^{2t} + 3e^{-4t}$, $f_2(t) = 3e^{2t} - 3e^{-4t}$

$f_1(t) = e^{2t} + 5e^{-4t}$, $f_2(t) = e^{2t} - e^{-4t}$

$f_1(t) = 5e^{2t} + e^{-4t}$, $f_2(t) = 5e^{2t} - 5e^{-4t}$

$f_1(t) = 6e^{2t}$, $f_2(t) = 0$

$f_1(t) = e^{2t}$, $f_2(t) = e^{-4t}$

Question 8

A polynomial is given by

$$P(Z) = Z^2 + aZ + b \, , \; a, b \in \mathbb{C}.$$

We are informed that $z_1 = 1 + i$ and $z_2 = 2i$ are roots in P. What are the values of a and b?

$a = -3i - 1 \, , \; b = -2 + 2i$

$a = 1 + i \, , \; b = 2i$

$a = 3i + 1 \, , \; b = 2 - 2i$

$a = 3i - 1 \, , \; b = -2 + 2i$

$a = -3i - 1 \, , \; b = -2 - 2i$

END OF THE EXAM