Homework Assignment 4

Solve the following problems without electronic aid. All answers must be justified, and intermediate steps should be provided to an appropriate extent.

a) Let V be the complex vector space $\mathbb{C}^{2\times 4}$ consisting of 2×4 matrices with complex coefficients. Does V have a complex subspace of dimension 9?

b) Let W be spanned by the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 0 \\ 4 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ -7 \\ -5 \\ 6 \end{bmatrix}.$$

Provide a basis for W and compute $\dim_{\mathbb{R}}(W)$.

c) Let $L : \mathbb{C}^3 \to \mathbb{C}^2$ be defined as follows:

$$L\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad v_1, v_2, v_3 \in \mathbb{C}.$$

We are given the following ordered bases

$$\beta = \left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) \quad \text{for } \mathbb{C}^2 \text{ and } \quad \gamma = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) \quad \text{for } \mathbb{C}^3.$$

Determine the mapping matrix ${}_\beta[L]_\gamma$.

d) We are given the following matrix:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ -1 & 0 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

Determine the eigenvalues of the matrix as well as bases for the corresponding eigenspaces. Can the matrix be diagonalized? If not, explain why not. If yes, specify a matrix $\mathbf{Q}$ and a diagonal matrix $\mathbf{\Lambda}$ such that $\mathbf{Q}^{-1} \cdot \mathbf{A} \cdot \mathbf{Q} = \mathbf{\Lambda}$.

e) 1. Let V denote the real vector space $\mathbb{R}^{2\times 2}$ consisting of 2×2 matrices with real coefficients. The following ordered basis is chosen for V:

$$\beta = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right).$$

We are given a linear map $M : V \to V$ defined by

$$M(\mathbf{A}) = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} \cdot \mathbf{A}, \quad \mathbf{A} \in V.$$

Determine the mapping matrix ${}_\beta[M]_\beta$.

2. Calculate the dimensions of the vector spaces $\ker(M)$ and $\text{image}(M)$. Hint: the mapping matrix ${}_\beta[M]_\beta$ can be of use here.

Your solution is to be uploaded as a pdf to the course's **DTU Learn module** under "Assignments". The deadline is **Saturday November 30 at 23:55**.