Homework Assignment 1

Solve the following problems without electronic aid:

a) Determine whether the following two logical propositions are logically equivalent:

$$(P \wedge Q) \Rightarrow R \quad \text{and} \quad \neg P \vee (Q \Rightarrow R).$$

b) State all real numbers x that fulfill the equation $|x-1|^2 = x^2 + |x|$.

c) A function $f : \mathbb{R} \to \mathbb{R}$ is given by the expression

$$f(x) = 4x^2 + 4|x-1|.$$

1. Compute the image set of the function.
2. Determine whether the function is injective.

d) For a complex number z we are given that $\operatorname{Arg}(z) = -\pi/3$ and $|z| = 2$.

1. Compute the polar coordinates of the complex number z^7.
2. Write the number z^7 in rectangular form.

e) Solve the binomial equation $z^3 = i$. The answers should be given in rectangular form as well as drawn in the complex plane.

f) As usual the principal argument of a complex number z is denoted by $\operatorname{Arg}(z)$. Determine whether the following logical propositions are true:

1. $\operatorname{Im}(z) > 0 \Rightarrow \operatorname{Arg}(z) > 0$.
2. $\operatorname{Arg}(z) \leq 0 \Rightarrow \operatorname{Im}(z) \leq 0$.
3. $\operatorname{Im}(z) = 0 \Rightarrow \operatorname{Arg}(z) = 0$.

Your solution must be uploaded as a pdf file to the course's **DTU Learn** module under "Assignments". The deadline is **Sunday September 29 at 23:55**.