1a) $(P \Leftrightarrow R) \vee Q$

P	Q	R	$P \Leftrightarrow R$	$(P \Leftrightarrow R) \vee Q$
T	T	T	T	T
F	T	T	F	T
T	F	T	T	T
F	F	T	F	F
T	T	F	F	T
F	T	F	T	T
T	F	F	F	F
F	F	F	T	T

1b) $(\neg P \wedge Q) \Rightarrow (P \vee Q)$

P	Q	$\neg P \wedge Q$	$(P \vee Q)$	$(\neg P \wedge Q) \Rightarrow (P \vee Q)$
T	T	F	T	T
F	T	T	T	T
T	F	F	T	T
F	F	F	F	T

It is a tautology.

2) $z^4 = -4$

$$-4 = 4 \cdot e^{i\pi}$$

Hence in polar form (not using the principal argument), the solutions are:

$$\sqrt{2} \cdot e^{i\frac{\pi}{4}}, \quad \sqrt{2}\, e^{i\frac{3\pi}{4}}, \quad \sqrt{2}\, e^{i\frac{5\pi}{4}}, \quad \sqrt{2}\, e^{i\frac{7\pi}{4}}$$

1^{st} quadrant $\quad$ 2^{nd} quadr. $\quad$ 3^{rd} quadr. $\quad$ 4^{th} quad

Hence the solution in the first quadrant is:

$$\sqrt{2} \cdot e^{i\frac{\pi}{4}} = \sqrt{2}\left(\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}\sqrt{2} \cdot i\right)$$

$$= \underline{1 + i}$$

3) $t_n = \sum_{k=2}^{n} \frac{1}{(k-1)k}$

a) $t_2 = \frac{1}{(2-1)\cdot 2} = \frac{1}{2}$

$t_3 = \frac{1}{2} + \frac{1}{(3-1)\cdot 3} = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

b) Basis $t=2$ $\quad \frac{1}{2} = \frac{2-1}{2}$ holds.

Step Assume: $t_{n-1} = \frac{n-2}{n-1}$ for some $n \geq 3$.

Then $t_n = t_{n-1} + \frac{1}{(n-1)\cdot n} = \frac{n-2}{n-1} + \frac{1}{(n-1)\cdot n}$

$= \frac{(n-2)\cdot n + 1}{(n-1)\cdot n} = \frac{n^2 - 2n + 1}{(n-1)\cdot n} = \frac{(n-1)^2}{(n-1)n}$

$= \frac{n-1}{n}$.

4)

a) $\begin{bmatrix} 1 & 4 & 2 \\ -3 & 1 & 7 \\ 2 & 1 & -3 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 2R_1]{R_2 \leftarrow R_2 + 3R_1} \begin{bmatrix} 1 & 4 & 2 \\ 0 & 13 & 13 \\ 0 & -7 & -7 \end{bmatrix}$

$\xrightarrow[R_3 \leftarrow -\frac{1}{7}R_3]{R_2 \leftarrow \frac{1}{13}R_2} \begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

pivots

A basis for W is $\{v_1, v_2\}$

b) $v_4 := \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is not in W, since v_1, v_2, v_4 are linearly independent:

$\begin{bmatrix} 1 & 4 & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 1 \\ 0 & 13 & 0 \\ 0 & -7 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -7 & -1 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5) $e = \{e_1, e_2, e_3\}$ standard basis

$${}_e[id]_\gamma = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

inverse

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & & \\ 2 & 0 & 0 & & 1 & \\ 0 & 1 & 1 & & & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{4} & 1 \end{array}\right]$$

Hence $${}_\gamma[id]_e = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ -\frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix}$$

Therefore

$$
{}_{e}[L]_{e} = {}_{e}[id]_{\gamma} \cdot {}_{\gamma}[L]_{\gamma} \cdot {}_{\gamma}[id]_{e}
$$

$$
= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ -\frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 7 & 2 & 1 \\ 6 & 0 & 2 \\ 3 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ -\frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \frac{1}{2} & \frac{13}{4} & 1 \\ -1 & \frac{7}{2} & 2 \\ \frac{1}{2} & \frac{5}{4} & 0 \end{bmatrix}
$$

6)

a) inhomogeneous

b) $z^2 - 6z + 9$ has roots $3, 3$

(a double root)

The general solution is therefore:

$$f(t) = c_1 e^{3t} + c_2 \cdot t \cdot e^{3t} + e^t$$