1MC

Expanding from column 1:

$$\det(\underline{A}) = -(-1)\det\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} + 1 \cdot \det\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} = -7 \quad (1e)$$

2MC

$$3f''(t) - 6f'(t) + 15f(t) = 0 \iff f''(t) - 2f'(t) + 5f(t) = 0$$

Characteristic polynomial:

$$p(\lambda) = \lambda^2 - 2\lambda + 5 \qquad p(\lambda) = 0: \ \lambda^2 - 2\lambda + 5 = 0$$

$$\lambda_1 = 1+2i, \ \lambda_2 = 1-2i$$

$$f(t) = c_1 e^t \cos 2t + c_2 e^t \sin 2t \quad (2b)$$

3MC

$$\begin{array}{l}
z+1 \,|\, z^3 - 2z^2 - z + 7 \,|\, \underline{z^2 - 3z} + 2 \\
\quad z^3 + z^2 \\
\hline
\quad\quad -3z^2 - z \\
\quad\quad -3z^2 - 3z \\
\hline
\quad\quad\quad 2z + 7 \\
\quad\quad\quad 2z + 2 \\
\hline
\quad\quad\quad\quad 5
\end{array} \quad (3d)$$

Alternatively:

$$P(z) = d(z) \cdot q(z) + r(z) \qquad d(z) = z + 1$$

$$P(-1) = 0 \cdot q(z) + r(z) = 5$$

4MC

4a) Neither surjective or injective

4b) Not injective

4c) Not surjective

4d) Not injective

4e) Not injective

4f) Bijective (correct)

5MC

Eigenvectors:

$$\begin{bmatrix} 3-i & -2 \\ 5 & -3-i \end{bmatrix} \rightarrow \begin{bmatrix} 3-i & -2 \\ 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_1 &= t \\ x_2 &= \frac{(3-i)}{2} \cdot t \end{aligned}$$

$$\underline{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3-i \end{pmatrix} \ (5b)$$

6MC

Eigenvalues of $\underline{A}$: $\lambda_1 = -1$, $\lambda_2 = -2$

Corresponding eigenvectors: $\underline{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\underline{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

General solution:

$$\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-2t} \quad , c_1, c_2 \in \mathbb{C}$$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix} = \begin{bmatrix} c_1 - 2c_2 \\ c_2 \end{bmatrix} \Rightarrow \begin{aligned} c_2 &= 4 \\ c_1 &= 11 \end{aligned}$$

$$\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} = \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ 4e^{-2t} \end{bmatrix} \ (6d)$$

7MC

$f(1) = 0$, $f(2) = 1$, $f(3) = 2f(2) + (f(1))^2 = 2$

$f(4) = 2f(3) + (f(2))^2 = 5$

$f(5) = 2f(4) + (f(3))^2 = 14$ (7c)

8MC

$$[L(1+z+z^2)]_\gamma = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$L(1+z+z^2) = 0 \cdot 1 + 4 \cdot z = 4z \ (8d)$$