Homework Assignment 2

Solve the following problems without electronic aid. All answers must be justified, and intermediate steps should be provided to an appropriate extent.

a) Compute all complex solutions to the equation $e^{2z} = 2 + i$.

b) We are given two complex numbers z_1 and z_2. We are informed that $\mathrm{Arg}(z_1) = \pi/4$ and $\mathrm{Arg}(z_2) = 3$. Determine $\mathrm{Arg}(-2z_1^4/z_2^{10})$.

c) Determine using the division algorithm whether the polynomial $Z^2 - 3Z + 2$ is a divisor of the polynomial $Z^5 - 3Z^4 + Z^3 + 4$.

d) 1. Show that the number -3 is a root in the polynomial $Z^3 - Z^2 + 36$. What is the multiplicity of this root?

2. Determine all roots in $Z^3 - Z^2 + 36$.

e) A function $f : \mathbb{N} \to \mathbb{Z}$ fulfills

$$f(n) \;=\; \begin{cases} 2 & \text{if } n = 1 \\ f(n-1)^2 - (n-1)^2 & \text{if } n \geq 2. \end{cases}$$

Compute $f(n)$ for $n \in \{1, 2, 3, 4, 5\}$.

f) Let r and s be two different complex numbers. Show by induction on n that

$$r^n + r^{n-1} \cdot s + r^{n-2} \cdot s^2 + \cdots + r \cdot s^{n-1} + s^n = \frac{r^{n+1} - s^{n+1}}{r - s}$$

for all $n \in \mathbb{N}$.

Your solution is to be uploaded as a pdf to the course's **DTU Learn** module under "Assignments". The deadline is **Sunday October 27 at 23:55**.