Exam 01003 E24

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål
Hvert rigtigt svar giver 1 point
Hvert forkert svar giver 0 point (der benyttes IKKE negative point)

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question
Every correct answer corresponds to 1 point
Every incorrect answer corresponds to 0 points (incorrect answers do not result in subtraction of points)

We are given the quadratic equation:

$$3z^2 - 6z + 12 = 0.$$

Which of the following complex numbers is a solution to the equation?

Vælg en svarmulighed

- $z = -6e^{12i}$
- None of the shown answers is a solution to the equation.
- $z = 2e^{-\frac{\pi}{3}i}$
- $z = 2e^{\pi i}$
- $z = 3e^{\frac{\pi}{3}i}$
- $z = 12e^{-\frac{\pi}{2}i}$
- $z = 3e^{\frac{\pi}{2}i}$

Let $p(Z)$ be a polynomial of degree 4.

$$p(Z) = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + a_4 Z^4$$

We are given the following information:
1) All of the coefficients a_0, a_1, a_2, a_3, a_4 are real.
2) 2 is a root of $p(Z)$ with a multiplicity of 2.
3) $1 + i$ is a root of $p(Z)$.
4) $p(1) = 7$.

What are the values of a_0 and a_3?

Vælg en svarmulighed

- $a_0 = -6, a_3 = 8$
- $a_0 = 56, a_3 = -42$
- $a_0 = 2, a_3 = 1 + i$
- $a_0 = 2, a_3 = 7$
- $a_0 = 7, a_3 = -6$
- $a_0 = -77, a_3 = 33$
- None of the provided answer options are correct.

Let the function $f : \mathbb{N} \to \mathbb{N}$ be given by:

$$f(n) = \begin{cases} 1 & \text{for } n = 1 \\ 2 & \text{for } n = 2 \\ 3f(n-1) - f(n-2) & \text{for } n \geq 3 \end{cases}$$

What is the value of $f(5)$?

Vælg en svarmulighed

- $f(5) = 5$
- None of the provided answer options are correct.
- $f(5) = 26$
- $f(5) = 1$
- $f(5) = 15$
- $f(5) = 13$
- $f(5) = 34$

Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map of the form:

$$L(\mathbf{v}) = \mathbf{A}\mathbf{v}, \text{ where } \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ is a real } 3 \times 3 \text{ matrix.}$$

We are being informed that:

$$L(\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}) = -2 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \ , \ L(\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}) = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \ ,$$
$$L(\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}) = 3 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Which of the below options correctly states two of the elements in the matrix $\mathbf{A}$?

Vælg en svarmulighed

- $a_{11} = 1 \ , \ a_{32} = 1$
- $a_{11} = -\frac{1}{2} \ , \ a_{32} = -5$
- $a_{11} = -2 \ , \ a_{32} = 3$
- $a_{11} = \frac{5}{2} \ , \ a_{32} = 2$
- $a_{11} = \frac{17}{2} \ , \ a_{32} = -\frac{3}{2}$
- $a_{11} = \frac{1}{2} \ , \ a_{32} = \frac{3}{2}$
- None of the provided answer options are correct.

Let $p(t) = at + b$, where $a, b \in \mathbb{R}$, be a first-degree polynomial.
Consider a second-order differential equation of the form:

$$f''(t) + 2f'(t) - 8f(t) = p(t)$$

We are being informed that $f_0(t) = -t + 5$ is a particular solution to the differential equation.
Which of the below options states a particular solution to the differential equation, $f_p(t)$, along with the values of a and b?

Vælg en svarmulighed

- $f_p(t) = -t + 5 \;,\; a = -1 \;,\; b = 5$
- None of the provided answer options are correct.
- $f_p(t) = 2e^{2t} \;,\; a = 8 \;,\; b = -42$
- $f_p(t) = e^{2t} + e^{-4t} - t + 5 \;,\; a = -1 \;,\; b = 5$
- $f_p(t) = e^{-2t} - t + 5 \;,\; a = -1 \;,\; b = 5$
- $f_p(t) = 4e^{4t} - t + 5 \;,\; a = 8 \;,\; b = -42$
- $f_p(t) = e^{-4t} - t + 5 \;,\; a = 8 \;,\; b = -42$

We are given the matrix $\mathbf{A} \in \mathbb{R}^{\mathbf{3} \times \mathbf{3}}$.

$$\mathbf{A} = \begin{bmatrix} -1 & 4 & 4 \\ 0 & 7 & 8 \\ 0 & -4 & -5 \end{bmatrix}$$

Which of the following vectors is not an egenvector of the matrix?

Vælg en svarmulighed

- $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$
- $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$
- $\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$
- All shown vectors are eigenvectors of matrix $\mathbf{A}$.

Let $\mathbf{A} \in \mathbb{C}^{\mathbf{4}\times\mathbf{4}}$ be given by:

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 4 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Which of the below numbers states the following determinant:

$D = \det((\mathbf{A}^{\mathbf{T}} + \mathbf{A}^{\mathbf{2}}) \cdot \mathbf{A}^{-\mathbf{1}})$?

Vælg en svarmulighed

- $D = 203$
- $D = -110$
- $D = 103$
- $D = 20$
- None of the provided answer options are correct.
- $D = 161$
- $D = -20$

Consider the real system of equations:

$$x_1 + x_2 + 3x_3 + x_4 = 7$$
$$-x_1 + x_3 = -5$$

Which of the below options states the general solution to the system of equations?

Vælg en svarmulighed

- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 0 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} 1 \\ -4 \\ 1 \\ 0 \end{pmatrix} \quad , \quad t_1, t_2 \in \mathbb{R}$
- None of the provided answer options state the general solution.
- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t_1 \begin{pmatrix} 5 \\ 2 \\ 0 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + t_3 \begin{pmatrix} 1 \\ -4 \\ 1 \\ 0 \end{pmatrix} \quad , \quad t_1, t_2, t_3 \in \mathbb{R}$
- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \\ 2 \end{pmatrix}$
- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 0 \\ 1 \end{pmatrix} + t_1 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \quad , \quad t_1 \in \mathbb{R}$
- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ 0 \\ 0 \end{pmatrix} + t_1 \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} + t_2 \begin{pmatrix} 1 \\ -4 \\ 1 \\ 0 \end{pmatrix} \quad , \quad t_1, t_2 \in \mathbb{R}$
- $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t_1 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} -1 \\ 4 \\ -1 \\ 0 \end{pmatrix} \quad , \quad t_1, t_2 \in \mathbb{R}$