Homework Assignment 3

Solve the following problems without electronic aid. All answers must be justified, and intermediate steps should be provided to an appropriate extent.

a) Compute the rank of the following matrix:

$$\begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

b) We are given a homogeneous system of linear equations over $\mathbb{R}$ with four equations in three unknowns. We are informed that the two vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

are solutions to the system.

1. Can it be determined whether the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$ is a solution to the system?

2. Can it be determined whether the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix}$ is a solution to the system?

c) Determine whether the following 3 vectors in $\mathbb{R}^4$ are linearly independent:

$$\begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ 5 \\ -3 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ -1 \end{bmatrix}.$$

d) Let n be a natural number and let $\mathbf{A}$ and $\mathbf{B}$ be two $n \times n$ matrices.

Show the identity $((\mathbf{A}+\mathbf{B})^2)^T = (\mathbf{A}^T + \mathbf{B}^T)^2$.

The problem sheet continues on the next page.

e) Compute the determinant of the following matrix:

$$\begin{bmatrix} 2 & 1 & 0 & -1 \\ -2 & 0 & -1 & 1 \\ 2 & 1 & 1 & -1 \\ -2 & 0 & 1 & 0 \end{bmatrix}.$$

f) Let n be a natural number and let $\mathbf{A}$ be an $n \times n$ matrix of the form:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \dots & 0 & \lambda_1 \\ 0 & \dots & 0 & \lambda_2 & 0 \\ 0 & \dots & \lambda_3 & 0 & 0 \\ \vdots & \iddots & & & \\ \lambda_n & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Such a matrix is called an antidiagional matrix.

1. Find the determinant for $n = 2, 3,$ and 4.

2. Show for all natural numbers $n \in \mathbb{Z}_{\geq 2}$ that

$$\det(\mathbf{A}) = (-1)^{\frac{n(n+3)}{2}} \lambda_1 \cdot \dots \cdot \lambda_n.$$

Hint: Use induktion on n.

Your solution is to be uploaded as a pdf to the course's **DTU Learn module** under "Assignments". The deadline is **Sunday November 10 at 23:55**.